1. Let $f(x)=e^{-2 x}$. Answer the following questions. Show all supporting work!!
a. Find the Taylor polynomial of order four for $f(x)$ at $x=0$.
b. Use the approximation from part "a" to approximate $f(0.2)$
c. Find the error bound for your approximation if $|x| \leq 0.2$.
2. Let $f(x)=2 \sin x$. Answer the following questions. Show all supporting work!!
a. Find the Taylor polynomial of order five for $f(x)$ at $x=0$.
b. Find the Lagrange Error Bound if $|x| \leq 0.1$.
3. Let $f(x)$ be a continuous, differentiable function such that $f(3)=1, f^{\prime}(3)=3, f^{\prime \prime}(3)=7, f^{\prime \prime \prime}(3)=5$.
a. Write a $3^{\text {rd }}$ order Taylor polynomial for $f(x)$ about $3(\mathrm{a}=3)$
b. Use the polynomial to approximate $f(2.9)$
c. Given $f^{(4)}(x) \leq 6$ for all x , find the Lagrange error bound for the approximation
4. Let $f(x)$ be a function with a $3^{\text {rd }}$ degree Taylor polynomial about $\mathrm{x}=2$ given by $P(x)=7-9(x-2)^{2}-3(x-2)^{3}$
a. Given $f^{(4)}(x) \leq 6$ for all x on $[0,4]$, find the Lagrange error bound for the approximation.
b. Give a convincing argument as to why $f(0)$ must be negative.
5. For $f(x)=\sin x$, determine the order of the Taylor polynomial about $\mathrm{x}=0$ needed to approximate $\sin (0.3)$ with an error of less than 0.001 .
